LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2015**

MT 4815 - ADVANCED GRAPH THEORY

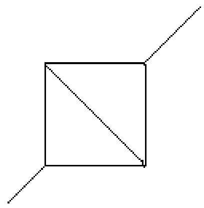
Date : 17/04/2015 Dept. No.   Max. : 100 Marks
Time : 09:00-12:00

Answer all questions. Each question carries 20 marks.

1. (a) Show that there exists no simple graphs corresponding to the following degree sequences (2, 2, 3, 4, 5, 5) and (2, 2, 4, 6). **(5)**

(OR)

(b) Find the number of spanning trees of the following graph G.



(5)

(c) (i) Let G be a disconnected graph with n vertices where n is even. If G has two components each of which is complete, prove that G has a minimum of $\frac{n(n-2)}{4}$ edges.

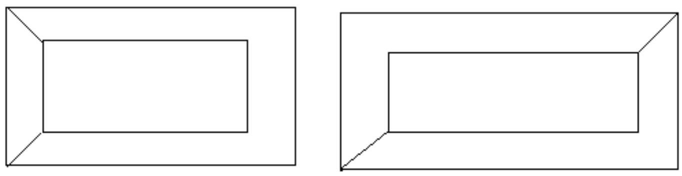
(ii) Prove a graph with 7 vertices cannot be isomorphic to its complement.

(iii) Let G be a graph with 21 edges,3 vertices of degree 4 and other vertices are of degree
3. Find the number of vertices of G. **(7 +4 +4)**

(OR)

(d) (i) Prove that a graph is bipartite if and only if it contains no odd cycle.

(ii) Determine whether the following graphs are isomorphic.



(9 + 6)

2. (a) Prove that a simple graph is Hamiltonian if and only if its closure is Hamiltonian. **(5)**

(OR)

(b) Prove that every tree has a centre containing one vertex or two adjacent vertices. **(5)**

(c) (i)State and prove Dirac theorem for Hamiltonian graphs.

(ii) Write Kruskal's algorithm. **(10 +5)**

(OR)

(d) (i) Prove that a graph G with $\nu \geq 3$ is 2- connected if and only if any two vertices of G are connected by at least two internally disjoint paths.

(ii) With usual notations prove that $\kappa \leq \kappa' \leq \delta$ **(8 +7)**

3. (a) Show that every 3-regular graph without cut edges has a perfect matching. **(5)**

(OR)

(b) Prove that $\chi' = \Delta$ for a bipartite graph G. **(5)**

(c) (i) State and prove the theorem that gives the necessary and sufficient condition for the existence of matching in a bipartite graph.

(ii) Show that K_{2n} is 1-factorable. **(10+5)**

(OR)

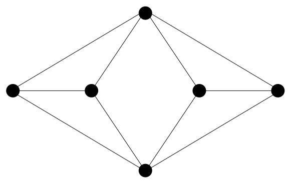
(d) Show that a bipartite graph G has a perfect matching if and only if $|N(S)| \geq |S|$ for all $S \subseteq V$. **(15)**

4. (a) (i) Define a covering of a graph G.

(ii) State and prove the theorem that relates independence number and covering number. **(2+3)**

(OR)

(b) Define a chromatic polynomial for a graph G. Find the chromatic polynomial for the following graph:



(5)

(c) State and prove Brook's theorem. **(15)**

(OR)

(d) (i) Prove that $\chi(G) \leq 1 + max\delta(H)$, where maximum is taken over all induced subgraphs H of G.

(ii). State and prove the theorem by Dirac. **(5 + 10)**

5. (a) Prove that K_5 is not planar. **(5)**

(OR)

(b) Let G be a nonplanar connected graph that contains no subdivision of $K_{3,3}$ or K_5 having a few edges. Then prove that G is simple and 3-connected. **(5)**

(c) (i) State and prove Five color theorem.

(ii) If G is a simple planar graph, prove that $\delta \leq 5$. **(10 + 5)**

(OR)

(d) Prove the theorem that gives the characterization of planar graphs. **(15)**

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